Math 20550 - Calculus III
Exam 3 Review

New things to know:

15. Multiple Integrals

15.4. **Double Integrals in Polar Coordinates.**

- Know the equations for converting between polar: (r, θ), and cartesian: (x, y), coordinates.
- Know how to write planar regions in polar coordinates.
- Know how to change a double integral from cartesian coordinates to polar coordinates.
  - Don't forget the r in $dA = r\,dr d\theta$!!

15.5. **Applications of Double Integrals.**

- Given the density function for a lamina, know how to find the:
  - mass of the lamina: m
  - moments of mass of the lamina about the $x-$ and $y-$axes: M_x and M_y
  - the coordinates for the center of mass $(\bar{x}, \bar{y})$ for the lamina
- It is really valuable to be able to compute double (and triple integrals) using symmetries if you can.

15.7. **Triple Integrals.**

- Be able to set up the bounds on and compute triple integrals in any order.
- Be able to read the region of integration off of the bounds on a triple integral.
- Be able to switch the order of integration in triple integrals.
- Given the density function for a solid, know how to find the:
  - mass of the solid: m
  - moments of mass about the $xy-$, $xz-$, and $yz-$ planes of the solid: M_{xy}, M_{xz}, and M_{yz}
  - the coordinates for the center of mass $(\bar{x}, \bar{y}, \bar{z})$ for the solid

15.8. **Triple Integrals in Cylindrical Coordinates.**

- Know the equations to converte between cylindrical coordinates: (r, θ, z), and cartesian coordinates: (x, y, z).
- Know how to write down the equations of surfaces in cylindrical coordinates
- Know how to convert a triple integral from cartesian coordinates to one in cylindrical coordintes
  - Don't forget the r in $dV = r\,dr d\theta dz$!!

15.9. **Triple Integrals in Spherical Coordinates.**

- Know the equations to converte between spherical coordinates: (ρ, θ, φ), and cartesian coordinates: (x, y, z).
- Know how to write down the equations of surfaces in spherical coordinates
- Know how to convert a triple integral from cartesian coordinates to one in spherical coordintes
  - Don't forget the $\rho^2 \sin\varphi$ in $dV = \rho^2 \sin\varphi\, d\rho d\theta d\varphi$!!

15.10. **Change of Variables in Multiple Integrals.**

- Know how to find the Jacobian, $\dfrac{\partial(x, y)}{\partial(u, v)}$, of a change of coordinates $x = x(u, v)$, $y = y(u, v)$. Likewise, know how to find the Jacobian for three variables as well.
- Know how to use the change of variables forumla to compute a double integral.
- Know how to choose a change of coordinates to make a region easier to integrate over.
- Know how to choose a change of coordinates to make a function easier to integrate.

16. Vector Calculus

16.1. Vector Fields.

- Given a vector field $\mathbf{F}$, know how to sketch a plot of the vector field.
- Know how to find the gradient vector field of a function.

16.2. Line Integrals.

- Know how to compute line integrals of a function f along a curve C with respect to:
  - arc length: $\int_C f\,ds$ (scalar line integral)
  - x: $\int_C f\,dx$
  - y: $\int_C f\,dy$
  - z: $\int_C f\,dz$
- Given the linear density function for a thin wire bent in the shape of a curve C, know how to find the mass of the wire as well as the coordinates for its center of mass.
- Know how to compute line integrals along a curve C which are a combination of line integrals with respect to x, y, and/or z.
- Know how to compute the vector line integral: $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$.
- Know that vetor line integrals give the work done by a force field $\mathbf{F}$ in moving a particle from one end of the curve C to the other.
- Know how to determine whether a vector line integral, $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ will be positive, negative, or zero given a plot of the vector field $\mathbf{F}$ with the curve C drawn in, c.f., exercises 17 and 18 in section 16.2. (Hint: it helps to remember the definition of the vector line integral which utilizes the unit tangent vector $\mathbf{T}$ of the curve.)
- Know what the orientation of a curve is, and how it affects line integrals (in particular, line integrals with respect to x, y, or z; or vector line integrals).

16.3. The Fundamental Theorem for Line Integrals (and a piece of section 16.5).

- Know the fundamental theorem for line integrals.
- Know what it means for a vector line integral to be independent of path.
- Know when a vector line integral is independent of path.
- Know what it means for a vector field to be conservative (2 and 3 variables).
- Know how to check whether a vector field is conservative (2 and 3 variables).
- Know how to find the potential function for a conservative vector field (2 and 3 variables).

16.4. Green's Theorem.

- Know the statement of Green's theorem.
- Be able to apply Green's theorem to compute integrals of the form $\displaystyle\oint_C P\,dx + Q\,dy$.